MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

(*Candidates are allowed additional 15 minutes for* ***only*** *reading the paper.*

They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions* ***EITHER*** *from* ***Section B OR Section C***

Section A: *Internal choice has been provided in three questions of four marks each and two questions of six marks each.*

Section B: *Internal choice has been provided in two questions of four marks each.*

Section C: *Internal choice has been provided in two questions of four marks each.*

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A (80 Marks)

Question 1 **[10×2]**

(i) If $f : R \rightarrow R$, $f(x) = x^3$ and $g : R \rightarrow R$, $g(x) = 2x^2+1$, and R is the set of real numbers, then find $fog\,(x)$ and $gof\,(x)$.

(ii) Solve: $\text{Sin}\,(2\tan^{-1}x) = 1$

(iii) Using determinants, find the values of k, if the area of triangle with vertices (−2, 0), (0, 4) and (0, k) is 4 square units.

(iv) Show that (A + A′) is symmetric matrix, if $A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$.

(v) $f(x) = \dfrac{x^2-9}{x-3}$ is not defined at $x = 3$. What value should be assigned to $f(3)$ for continuity of $f(x)$ at $x = 3$?

(vi) Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on R.

(vii) Evaluate: $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

(viii) Using L'Hospital's Rule, evaluate: $\lim\limits_{x \to 0} \frac{8^x - 4^x}{4x}$

(ix) Two balls are drawn from an urn containing 3 white, 5 red and 2 black balls, one by one without replacement. What is the probability that at least one ball is red?

(x) If events A and B are independent, such that P(A) $= \frac{3}{5}$, P(B) $= \frac{2}{3}$, find $P(A \cup B)$.

Question 2 **[4]**

If $f : A \rightarrow A$ and $A = R - \left\{ \frac{8}{5} \right\}$, show that the function $f(x) = \frac{8x+3}{5x-8}$ is one – one onto.

Hence, find f^{-1}.

Question 3 **[4]**

(a) Solve for x:

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\text{®}}{4}$$

OR

(b) If $\sec^{-1} x = \operatorname{cosec}^{-1} y$, show that $\frac{1}{x^2} + \frac{1}{y^2} = 1$

Question 4 **[4]**

Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

Question 5 **[4]**

(a) Show that the function $f(x) = |x-4|$, $x \in R$ is continuous, but not differentiable at $x = 4$.

OR

(b) Verify the Lagrange's mean value theorem for the function:

$$f(x) = x + \frac{1}{x} \text{ in the interval } [1, 3]$$

Question 6 **[4]**

If $y = e^{\sin^{-1}x}$ *and* $z = e^{-\cos^{-1}x}$, prove that $\dfrac{dy}{dz} = e^{\text{®}/2}$

Question 7 **[4]**

A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level. The bottom of the ladder is pulled away from the wall, along the ground, at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

Question 8 **[4]**

(a) Evaluate: $\int \frac{x(1+x^2)}{1+x^4}\,dx$

OR

(b) Evaluate: $\int_{-6}^{3} |x+3|\,dx$

Question 9 **[4]**

Solve the differential equation: $\dfrac{dy}{dx} = \dfrac{x+y+2}{2(x+y)-1}$

Question 10 **[4]**

Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5 black balls. Two balls are drawn from Bag A and placed in Bag B. Then, what is the probability of drawing a white ball from Bag B?

Question 11 **[6]**

Solve the following system of linear equations using matrix method:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9$$

$$\frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52$$

$$\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$$

Question 12 **[6]**

(a) The volume of a closed rectangular metal box with a square base is 4096 cm^3. The cost of polishing the outer surface of the box is ₹ 4 per cm^2. Find the dimensions of the box for the minimum cost of polishing it.

OR

(b) Find the point on the straight line $2x + 3y = 6$, which is closest to the origin.

Question 13 **[6]**

Evaluate: $\int_0^{\circledR} \frac{x\tan x}{\sec x+\tan x}dx$

Question 14 **[6]**

(a) Given three identical Boxes A, B and C, Box A contains 2 gold and 1 silver coin, Box B contains 1 gold and 2 silver coins and Box C contains 3 silver coins. A person chooses a Box at random and takes out a coin. If the coin drawn is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver.

OR

(b) Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$.

Also, find the probability of obtaining at most one success.

SECTION B (20 Marks)

Question 15 **[3×2]**

(a) If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}| = 13$ and $|\vec{a}| = 5$, find the value of $|\vec{b}|$.

(b) Find the length of the perpendicular from origin to the plane $\vec{r}.(3i - 4j - 12k) + 39 = 0$.

(c) Find the angle between the two lines $2x = 3y = -z$ and $6x = -y = -4z$.

Question 16 **[4]**

(a) If $\vec{a} = i - 2j + 3k$, $\vec{b} = 2i + 3j - 5k$, prove that $\vec{a}$ and $\vec{a} \times \vec{b}$ are perpendicular.

OR

(b) If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, find the value of x such that the vectors $\vec{\sim} = (x-2)\vec{a} + \vec{b}$ and $\vec{\ } = (3+2x)\vec{a} - 2\vec{b}$ are collinear.

Question 17 **[4]**

(a) Find the equation of the plane passing through the intersection of the planes $2x + 2y - 3z - 7 = 0$ and $2x + 5y + 3z - 9 = 0$ such that the intercepts made by the resulting plane on the x-axis and the z-axis are equal.

OR

(b) Find the equation of the lines passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

Question 18 **[6]**

Draw a rough sketch and find the area bounded by the curve $x^2 = y$ and $x + y = 2$.

SECTION C (20 Marks)

Question 19 **[3×2]**

(a) A company produces a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of ₹ 8 per unit. Find the break-even point.

(b) The total cost function for a production is given by $C(x) = \frac{3}{4}x^2 - 7x + 27$.

Find the number of units produced for which M.C. = A.C.
(M.C.= Marginal Cost and A.C. = Average Cost.)

(c) If $\overline{x} = 18, \overline{y} = 100, \pm_x = 14, \pm_y = 20$ and correlation coefficient $r_{xy} = 0 \cdot 8$, find the regression equation of y on x.

Question 20 **[4]**

(a) The following results were obtained with respect to two variables x and y:

$$\sum x = 15, \sum y = 25, \sum xy = 83, \sum x^2 = 55, \sum y^2 = 135 \text{ and } n = 5$$

(i) Find the regression coefficient b_{xy}.

(ii) Find the regression equation of x on y.

OR

(b) Find the equation of the regression line of y on x, if the observations (x, y) are as follows:

(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18)

Also, find the estimated value of y when $x = 14$.

Question 21 **[4]**

(a) The cost function of a product is given by $C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$ where x is the number of units produced. How many units should be produced to minimise the marginal cost?

OR

(b) The marginal cost function of x units of a product is given by MC = $3x^2 - 10x + 3$. The cost of producing one unit is ₹ 7. Find the total cost function and average cost function.

Question 22 **[6]**

A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹ 48 per unit and product B is sold for ₹ 40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum gross income?
Formulate the above as a Linear Programming Problem and solve it, indicating clearly the feasible region in the graph.
